

A Century-Old Question: Does a Crookes Paddle Wheel Cathode Ray Tube Demonstrate that Electrons Carry Momentum?

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In 1879, in the midst of the debate between English and continental scientists about the nature of cathode rays, William Crookes conducted an experiment in which a small mill or “paddle wheel” was pushed along tracks inside a cathode ray tube (CRT) (similar to that shown in Fig. 1) when connected to a high-voltage induction coil. Crookes attributed the motion of the wheel to momentum transfer from the cathode rays (electrons) to the wheel,¹ and interpreted the experiment as providing evidence that cathode rays were particles. In 1903 Thomson discounted Crookes’ interpretation by calculating that the rate of momentum transfer (which he estimated at no more than 2×10^{-3} dyn, equivalent to 2×10^{-8} N) would be far too small to account for the observed motion of the wheel,² instead attributing the motion to the radiometric effect. The misconception was not laid to rest, however, and despite an effort in 1961 to draw attention to Thomson’s original work and so remove the error from textbooks,³ the notion that a Crookes paddle wheel CRT demonstrates that electrons carry momentum continues to be taught in high school physics courses⁴ and

wheel. We then measured the actual acceleration of the wheel in the CRT by video analysis of its motion and determined the moment of inertia of the wheel along with its mass and dimensions. We could then compare the force, which really acts on the wheel to produce the observed motion to the maximum impulsive force that is supplied by the electrons. Our measurements yield a maximum impulsive force due to the electrons [$F_{el} = (1.1 \pm 0.3) \times 10^{-8}$ N], which is within a factor of two of Thomson’s estimate, and which is more than two orders of magnitude smaller than the force that is responsible for the observed acceleration of the paddle wheel [$F_W = (6 \pm 2) \times 10^{-6}$ N]. This means that the rotation of the wheel is certainly not due to transferred momentum from the electron beam, and the results of the experiment should not be taught to students as proof that electrons are particles with mass that carry momentum.

Theory

The electron beam. As noted by Thomson,² the maximum possible momentum a single electron with speed v can

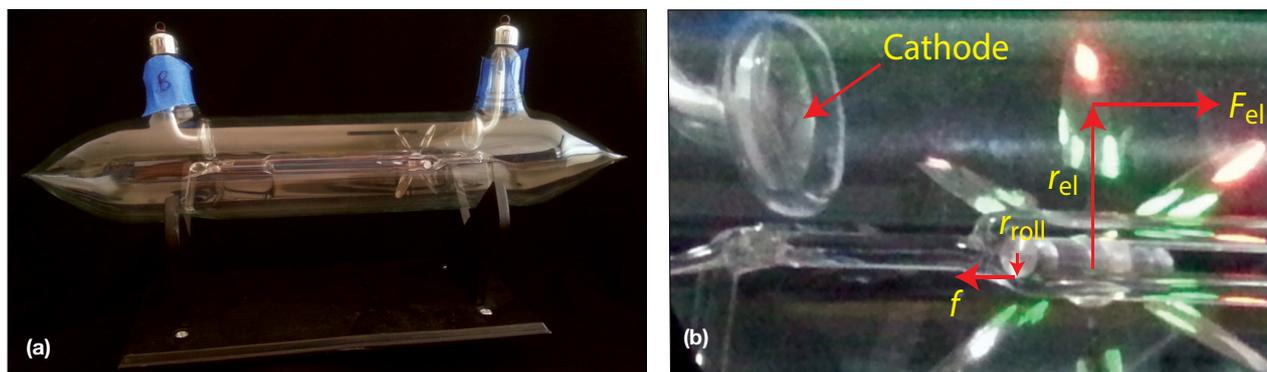


Fig. 1. (a) A paddle wheel CRT. (The label “B” on the left side allows us to distinguish orientations of the CRT during the experiment). (b) Close-up of the paddle wheel while illuminated by the electron beam. The measurements r_{roll} , r_{el} , the impulsive force the electrons exert on the wheel (F_{el}), and the force due to friction between the wheel and the tracks (f) are indicated.

is so widely accepted that even Abraham Pais⁵ recounts that Crookes “performed a series of elegant experiments in which he inserted such objects into his vacuum tube as...a tiny vaned paddle-wheel which served to demonstrate that the rays transmit momentum.” As the idea remains pervasive, and its repudiation is reliant upon a theoretical estimation by Thomson a century ago, we have performed an experimental test to resolve the question.

Our approach was to first measure the actual momentum of an electron in a CRT, and so determine an upper bound upon the impulsive force the beam can provide to the paddle

impart to the wheel is $2m_e v$, which would correspond to an elastic collision. The maximum possible impulsive force the electron beam could supply is then

$$F_{el} = 2m_e v \frac{i}{e}, \quad (1)$$

where i is the average current flowing through the CRT and e the charge on an electron.

The maximum power supplied by the electron beam to increase the mechanical energy of the paddle wheel is the average kinetic energy carried by each electron multiplied by the number of electrons hitting the wheel per second

$$P_{el} = \frac{m_e v^2}{2} \frac{i}{e}. \quad (2)$$

The paddle wheel. The motion of the paddle wheel can be analyzed by applying $\Sigma F = ma_{cm}$ and $\Sigma \tau = I\alpha$. As the wheel rolls without slipping along the track, $\alpha r_{roll} = a_{cm}$, where r_{roll} is the radius of the part of the wheel that makes contact with the rails and a_{cm} is the linear acceleration of the center of mass. Summing forces parallel to the track

$$F_w - f = ma_{cm}, \quad (3)$$

where F_w is the linear force that must act on the wheel to produce the observed acceleration a_{cm} and f is the frictional force the rail applies to the wheel. Summing torques we obtain

$$F_w r_{el} + f r_{roll} = I \frac{a_{cm}}{r_{roll}}, \quad (4)$$

where r_{el} is the average distance from the axis at which the linear force is applied to the wheel (effectively the center of the vanes as the electron beam illuminates the entire vane). Eliminating the frictional force f between the equations gives the actual force acting upon the vanes of the paddle wheel to cause a linear acceleration a_{cm}

$$F_w = \frac{(I + Mr_{roll}^2)}{r_{roll}^2} \frac{r_{roll}}{(r_{el} + r_{roll})} a_{cm}. \quad (5)$$

We have expressed the equation in this particular form since $r_{roll}^2 / (I + Mr_{roll}^2)$ is measured directly during the experiment.

The rate at which the paddle wheel gains mechanical energy when its velocity changes from v_i to v_f over time t is given by

$$\frac{\Delta KE}{t} = \frac{1}{2} \left(\frac{I + Mr_{roll}^2}{r_{roll}^2} \right) \frac{(v_f^2 - v_i^2)}{t}. \quad (6)$$

Method

Maximum possible impulsive force supplied by the electrons. This was determined by measuring both the momentum of an individual electron in the beam and the current flowing through the CRT.

The momentum of electrons was obtained by measuring the radius of curvature of the electron beam in a known magnetic field provided by a pair of Helmholtz coils, following Thomson's geometrical approach in Ref. 6. The CRT used for this part of the experiment was, by necessity, one containing a flat phosphor-coated plate along the length of the tube so that the beam was visible, rather than the paddle wheel tube. This tube was produced by the same manufacturer and excited by the same voltage (3 V) applied to a 50-kV induction coil as was used for powering the paddle wheel tube, so it is reasonable to expect the velocity of the electrons in the beam to be comparable.

The current through both the phosphor CRT and the paddle wheel tube was determined using a 50-MHz digital oscilloscope to measure the voltage across a 50-Ω resistor (to

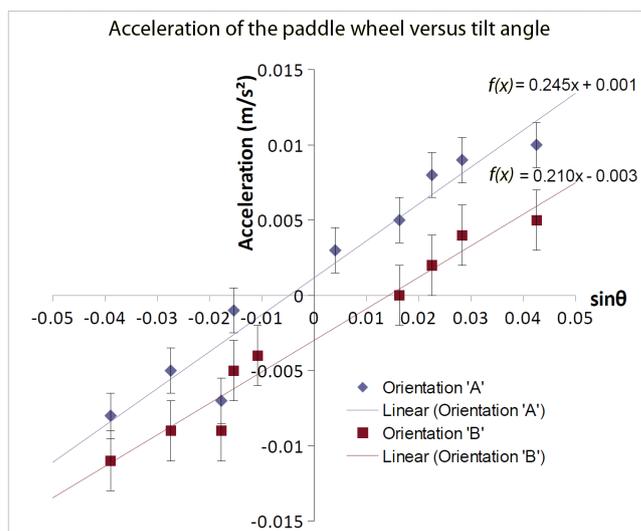


Fig. 2. Data for the acceleration of the wheel in response to a tilt of the CRT. Diamond markers: the wheel rolls from terminal A [behind the CRT in Fig. 1(a)] to B [visible in Fig. 1(a)]. Square markers: Opposite orientation. A linear least-squares fit is shown for each set.

impedance match the oscilloscope probe) in series with the CRT. The induction coil produced a current in both the paddle wheel and phosphor plate CRT that flowed in bursts with a duration of 750 μs every 33 ms. Each current burst began with a large amplitude that reduced to zero in a roughly linear fashion over the duration of the pulse. Superimposed on this behavior were higher frequency oscillations. The average voltage across the 50-Ω resistor (and thus the average current flowing) for each pulse was measured using the digital oscilloscope and found through averaging multiple measurements to be 45 ± 10 mV, giving an average current flowing of $i = [(45 \pm 10 \text{ mV}) \times (0.75/33)] / 50 \Omega = (2.0 \pm 0.4) \times 10^{-5}$ A.

The maximum impulsive force and the power supplied by the electron beam was then determined from Eqs. (1) and (2) and reported in Table I, along with the associated uncertainties. We note that the values for average current $i = (2.0 \pm 0.4) \times 10^{-5}$ A, and velocity of electrons $(5.0 \pm 0.3) \times 10^7$ ms⁻¹, obtained in our experiment are very close to those cited by Thomson as representative values ($i \approx 10^{-5}$ A and $v \approx 5 \times 10^7$ ms⁻¹ respectively).²

Actual linear force required to produce observed acceleration. There were several components to this measurement. The mass and the radius of the part of the wheel in contact with the tracks, r_{roll} [indicated in Fig. 1(b)], were measured directly (which necessitated breaking the CRT to access the wheel at the end of the other experiments) using a scale accurate to 0.01 g and a micrometer, and the results are shown in Table II. Then r_{el} [indicated in Fig. 1(b)] was taken to be the distance from the axis to the center of the vanes of the paddle wheel and was also measured with a micrometer. We used the relationship between the moment of inertia and the linear acceleration for an object rolling without slipping down a slope⁷ to obtain

$$a_{\text{cm}} = Mg \frac{r_{\text{roll}}^2}{Mr_{\text{roll}}^2 + I} \sin \theta. \quad (7)$$

Note that the gradient of this linear relationship (divided by the weight of the wheel Mg) appears in Eq. (5). To experimentally determine the coefficient of Eq. (7), we placed the paddle wheel tube on an adjustable slope (the CRT was *not* connected to the induction coil for this part of the experiment). The motion of the wheel in response to a range of gradients was videoed and analyzed using the free physics software Tracker⁸ to determine the acceleration of the wheel as a function of the tilt angle θ , which is shown in Fig. 2. These measurements were performed for two orientations of the CRT to account for the presence of a small internal slope in the rails relative to the body of the tube. This slope was such that it was “downhill” for the paddle wheel from the terminal marked B in Fig. 1(a) to the other terminal. We defined orientation B as having this terminal on the left, while in orientation A the tube was rotated 180° so terminal B is on the right. The average of the slope for the two data sets was taken and is shown in the results of Table II as the coefficient of Eq. (7), with the associated uncertainty taken as the standard error in the least-squares linear fit to the data. Note that the data sets for orientations A and B are vertically offset from each other due to the slope of the internal rails.

The linear acceleration of the paddle wheel while connected to the induction coil with the electron beam switched

Table I. Experimental results obtained for the electron beam.

Maximum impulsive force due to electrons		
Variable	Value	Uncertainty (%)
v	$(5.0 \pm 0.3) \times 10^7 \text{ ms}^{-1}$	5
i	$(2.0 \pm 0.4) \times 10^{-5} \text{ A}$	20
F_{el}	$(1.1 \pm 0.3) \times 10^{-8} \text{ N}$	25
P_{el}	$(0.14 \pm 0.04) \text{ W}$	30

Table II. Experimental results obtained for the motion of the paddle wheel.

Actual linear force acting on paddle wheel		
Measured variable	Value	Uncertainty (%)
M	$(7.0 \pm 0.2) \times 10^{-4} \text{ kg}$	3
r_{roll}	$(1.08 \pm 0.02) \times 10^{-3} \text{ m}$	2
r_{el}	$(1.16 \pm 0.01) \times 10^{-2} \text{ m}$	1
$\frac{Mg r_{\text{roll}}^2}{Mr_{\text{roll}}^2 + I}$	$(0.23 \pm 0.03) \text{ ms}^{-2}$	10
a_{cm} (with e beam)	$(2.3 \pm 0.5) \times 10^{-3} \text{ ms}^{-2}$	20
F_{W}	$(6 \pm 2) \times 10^{-6} \text{ N}$	40
$0.5(v_f^2 - v_i^2)/t$	$(4.5 \pm 0.9) \times 10^{-5} \text{ m}^2 \cdot \text{s}^{-3}$	20
P_{W}	$(1 \pm 0.3) \times 10^{-6} \text{ W}$	33

on was also measured by videoing the wheel and analyzing its motion using the Tracker software. Two measurements were taken, one in orientation A and one in orientation B, but always maintaining the terminal at end B as the cathode. These measurements were averaged to account for any residual slope in the table, which was leveled as far as possible. The experiment was also repeated with the electron beam switched off, to allow the small component of the acceleration of the wheel that was due to the slope of the rails inside the tube to be subtracted from the acceleration measured when the beam was on.

The force acting on the wheel to produce the observed linear acceleration of the wheel was then calculated using Eq. (5) along with the rate of change of mechanical energy of the wheel using Eq. (6). The percentage errors in the measured quantities were also summed and are reported in Table II.

Results and discussion

Our experimental results are contained in Fig. 2 and Tables I and II. We conclude that the impulsive force delivered by the momentum of the electron beam is not enough to cause the rotation of the paddle wheel since it is two orders of magnitude too small.

If it is not the momentum of the electrons that rotates the wheel, then what is responsible for its motion?

While it is true that electrons in the CRT carry little momentum due to the small mass of the electrons compared to atoms ($p = 9.11 \times 10^{-31} \text{ kg} \times 5 \times 10^7 \text{ ms}^{-1} = 5 \times 10^{-23} \text{ kg} \cdot \text{ms}^{-1}$ per electron), the *kinetic energy* carried by the electrons is substantial due to the fact they are moving at 15% of the speed of light ($KE = 1/2 mv^2 = 1 \times 10^{-15} \text{ J} = 7 \text{ keV}$ per electron—enough to cause the emission of x-rays when they collide with the glass at the end of the tube). Our measurements verified that the power supplied by the electron beam [$P_{\text{el}} = (0.14 \pm 0.04) \text{ W}$, in Table I] is several orders of magnitude greater than the mechanical energy gained by the paddle wheel per unit time [$P_{\text{W}} = (1 \pm 0.3) \times 10^{-6} \text{ W}$, in Table II]. As noted by Thomson, the most plausible explanation for the motion of the wheel would appear to be that the inelastic collisions between electrons and the wheel produces (very inefficient) transformation of heat energy into rotational motion via a type of radiometric effect (see Ref. 9, available online, for a thorough review of historical and modern understanding of the phenomena). The two mechanisms (due separately to Reynolds and Einstein⁹) by which a heat differential between opposite sides of the vanes can result in rotational motion both require the presence of some residual gas molecules in the tube. Thus the origin of the motion of the wheel could potentially be tested¹⁰ more rigorously by using a CRT in which the degree of vacuum can be adjusted (as the radiometric effect disappears if gas pressure is too low) and or where the ambient temperature could be varied (as a radiometer operates as a type of heat engine).

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